Improved Class of Ratio –type estimator for finite Population mean

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Abstract: In this paper, an improved class of Ratio type estimator has been proposed to estimate population mean of the characteristic under study. Particular cases of proposed estimator have been obtained which are improvement over existing estimator in literature and, thus, it serves as a unified study of several estimators. The expression for bias and mean square error of the proposed estimator including its particular cases have been derived up to the first order of approximation and compared Theoretically with the other improved ratio-type estimators and conditions found for which the proposed estimator is better than improved ratio-type estimators. An empirical study has also been carried out to demonstrate the efficiencies of proposed estimator.

Key words: Bias, Efficiency, Mean Square Error, Ratio-Type Estimators

I. Introduction

The aim of sampling is to obtain precise results about the population parameters of the study variables based on random samples. In sampling theory, the use of auxiliary information in the estimation of population values of study characteristic has been frequently acknowledged to increase the precision of the estimator. 1] introduced the use of auxiliary information as classical ratio estimator. Later on , in literature , it has been shown by various authors viz [2], [3], [4],[5], [6] [7] and [8] that the bias and the mean square error of ratio estimator can be reduced with the application of transformation on auxiliary variable.

The fact that ratio and product estimator have superiority over sample mean estimator when the correlation between study characteristic and auxiliary variable in the population is either positively or negatively high, led the statistician to focus their attention on the modification of such conventional estimators so that the modified estimators can work efficiently even if the correlation is low. Such modified estimators are generally developed either using one or more unknown constants or introducing a convex linear combination of sample and population means of auxiliary characteristic with unknown weights. In both the cases, optimum choices of unknown parameters are made by minimizing the mean square error of modified estimators so that they become superior than the conventional one. In defining modified estimators based on unknown parameters ,actually a class of estimators were developed which include a number of classical estimators and, thus, enable to make a unified study of several estimators.

In the sequence of suggesting modification over classical ratio and product estimators [9] , [10] ,[11] , [12], and [13] considered a ratio –type estimators with the use of weighted mean of $\bar{X}$ and $\bar{x}$ in place of $\bar{x}$ in classical ratio and product estimators. [14], [15] and [16] did some other remarkable works in this direction.

Motivated by the work done by above mentioned authors , in the present paper an improved class of ratio-type estimator of population mean of study characteristic has been suggested which includes the various ratio –type estimators that are improvement over existing estimator in terms of biases and MSE's.

II. Notation

Let Y_i denotes the value of characteristic under study for the i[th] unit in population of size N (i=1,2…N). and X_i the value of auxiliary characteristic for the i[th] unit in population. Then

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$; The population mean of characteristic under study.

$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$; The population mean of auxiliary characteristic.

$S_Y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$; The population mean square of characteristic under study.

$S_X^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$; The population mean square of auxiliary characteristic.

$R = \frac{\bar{Y}}{\bar{X}}$;The ratio of population means.

Let a sample of size n has been drawn by method of simple random sampling without replacement. Then y_i denotes the values of characteristic under study which is

ISSN 2229-5518

included in the sample at i^th draw (i= 1,2, 3, . . . ,n). and x_i denotes the values of auxiliary characteristic which is drawn in the sample at i^th draw. Let us further denote

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$; The sample mean.

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$; The sample mean of auxiliary characteristic.

$\hat{R} = \frac{\bar{y}}{\bar{x}}$; The ratio of the sample mean.

$C_Y = \frac{S_Y}{\bar{Y}}$; The coefficient of variation of characteristic under study.

$C_X = \frac{S_X}{\bar{X}}$; The coefficient of variation of auxiliary characteristic.

$\rho = \frac{S_{XY}}{S_X S_Y}$; The Correlation coefficient between the value of auxiliary variable and value of characteristic under study.

III. Proposed Estimators:

To estimate the mean of characteristic under study, an improved class of ratio-type estimator has been proposed which is as follows:

$$\bar{y}_p = \bar{y}\left[\frac{\sqrt{\bar{X}}}{\alpha\sqrt{\bar{x}}+(1-\alpha)\sqrt{\bar{X}}}\right]^K \qquad (1)$$

where α and k are constants.

.IV. Bias and MSE of Proposed Estimators

To obtain bias and mean square error of proposed estimator up to first order of approximation, let us consider

$\bar{y} = \bar{Y}\,(1+ e_0)$ and $\bar{x} = \bar{X}\,(1+ e_1)$

Then clearly, we have

$E(e_0) = E(e_1) = 0$

$E(e_0)^2 = (\frac{1}{n} - \frac{1}{N})C_Y^2$.

$E(e_1)^2 = \left(\frac{1}{n} - \frac{1}{N}\right) C_X^2$.

$E(e_0 e_1) = \left(\frac{1}{n} - \frac{1}{N}\right) \rho C_X C_Y$.

Now, up to the first order of approximation the expected valve of $\bar{y}_p$ is obtained as

$$E(\bar{y}_p) = \bar{Y}\left[1 + \left(\frac{1}{n} - \frac{1}{N}\right)\left(\left(\frac{k(k+1)}{4}\alpha^2 + \frac{1}{8}\,k\alpha\right) C_X^2 - \frac{1}{2}k\alpha\rho C_X C_Y\right)\right]$$

Thus, Bias $(\bar{y}_p) = \bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)\left[\left(\frac{k(k+1)}{4}\alpha^2 + \frac{1}{8}\,k\alpha\right) C_X^2 - \frac{1}{2}k\alpha\rho C_X C_Y\right]$ (2)

And, MSE $(\bar{y}_p) = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_Y^2 + \frac{1}{4}\,k^2\alpha^2 C_X^2 - k\alpha\rho C_X C_Y\right)$ (3)

V. Optimum value of K

From mean square error, the optimum value of k is obtained as

$$k = \frac{2\rho C_Y}{\alpha C_X}$$

for optimum value of k the mean square error reduces to

$$\text{MSE}\,(\bar{y}_p)_{min} = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right) C_Y^2\,(1 - \rho^2) \qquad (5)$$

This is the mean square error of linear regression estimator.

VI. Particular cases of Proposed Estimator:

When k=1 and $\alpha = w$ then proposed estimator $\bar{y}_p$ reduces to an estimator which is an improvement over the estimator proposed by Walsh (1979) and becomes

$$\bar{y}_{\sqrt{w}} = \bar{y}\left[\frac{\sqrt{\bar{X}}}{w\sqrt{\bar{x}}+(1-w)\sqrt{\bar{X}}}\right] \qquad (6)\ (1)$$

The expression for Bias and M.S.E are

$$\text{Bias}(\bar{y}_{\sqrt{w}}) = \bar{Y}\frac{1}{2}w\left(\frac{1}{n} - \frac{1}{N}\right)\left[\left(w + \frac{1}{4}\right) C_X^2 - w\rho C_X C_Y\right] \qquad (7)$$

$$\text{MSE}\,(\bar{y}_{\sqrt{w}}) = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_Y^2 + \frac{1}{4}\,w^2 C_X^2 - w\rho C_X C_Y\right) \qquad (8)$$

When k = -1 and $\alpha = -\gamma$ then proposed estimator $\bar{y}_p$ reduces to an estimator which is an improvement over the estimator proposed by Ray et al (1979) and reduces to

$$\bar{y}_{\sqrt{RS}} = \bar{y}\left[\frac{(1+\gamma)\sqrt{\bar{X}} - \gamma\sqrt{\bar{x}}}{\sqrt{\bar{X}}}\right] \qquad (9)$$

The expression for Bias and M.S.E are

$$\text{Bias}(\bar{y}_{\sqrt{RS}}) = \frac{1}{2}\,\bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)\gamma\left(\frac{1}{4}C_X^2 - \rho C_X C_Y\right) \qquad (10)$$

$$\text{MSE}\,(\bar{y}_{\sqrt{RS}}) = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_Y^2 + \frac{1}{4}\,\gamma^2 C_X^2 - \gamma\rho C_X C_Y\right)$$

ISSN 2229-5518

(11)

When k = -1 and $\alpha = 1 - f$ then proposed estimator $\bar{y}_p$ reduces to an estimator which is an improvement over the estimator proposed by Srivenkataramana and tracy (1979) .Thus we have

$$\bar{y}_{\sqrt{ST}} = \bar{y}\left[f + \frac{(1-f)\sqrt{\bar{x}}}{\sqrt{\bar{X}}}\right] \quad (12)$$

The expression for Bias and M.S.E are

$$\text{Bias}(\bar{y}_{\sqrt{ST}}) = \frac{1}{2}\,\bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)(1-f)\left(\rho C_X C_Y - \frac{1}{4}C_X^2\right) \quad (13)$$

$$\text{MSE}\left(\bar{y}_{\sqrt{ST}}\right) = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_Y^2 + \frac{1}{4}(1-f)^2 C_X^2 - (1-f)\rho C_X C_Y\right) \quad (14)$$

When k = -1 and $\alpha = -\left(\frac{n}{N-n}\right)$ then proposed estimator $\bar{y}_p$ reduces to an estimator which is an improvement over the estimator proposed by Srivenkataramana (1980) and reduces to

$$\bar{y}_{\sqrt{S}} = \bar{y}\left[\frac{N\sqrt{\bar{X}} - n\sqrt{\bar{x}}}{(N-n)\sqrt{\bar{X}}}\right] \quad (15)$$

The expression for Bias and M.S.E are

$$\text{Bias}(\bar{y}_{\sqrt{S}}) = \frac{1}{2}\,\bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)\left(\frac{n}{N-n}\right)\left(\frac{1}{4}C_X^2 - \rho C_X C_Y\right) \quad (16)$$

$$\text{MSE}\left(\bar{y}_{\sqrt{S}}\right) = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_Y^2 + \frac{1}{4}\left(\frac{n}{N-n}\right)^2 C_X^2 + \left(\frac{n}{N-n}\right)\rho C_X C_Y\right) \quad (17)$$

When k = -1 and $\alpha = 1 - w$ then proposed estimator $\bar{y}_p$ reduces to an estimator which is an improvement over the estimator proposed by Srivenkataramana (1980) .Thus it reduces to

$$\bar{y}_{\sqrt{V}} = \bar{y}\left[\frac{w\sqrt{\bar{X}} + (1-w)\sqrt{\bar{x}}}{\sqrt{\bar{X}}}\right] \quad (18)$$

The expression for Bias and M.S.E are

$$\text{Bias}(\bar{y}_{\sqrt{V}}) = \frac{1}{2}\,\bar{Y}\left(\frac{1}{n} - \frac{1}{N}\right)(1-w)\left(\rho C_X C_Y - \frac{1}{4}C_X^2\right) \quad (19)$$

$$\text{MSE}\left(\bar{y}_{\sqrt{V}}\right) = \bar{Y}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_Y^2 + \frac{1}{4}(1-w)^2 C_X^2 + (1-w)\rho C_X C_Y\right). \quad (20)$$

VII. Comparison of proposed Estimator with Existing Estimators

In this section, conditions have been derived under which proposed class of estimator is more efficient than existing ratio-type estimators.

First proposed class of estimator is compared with walsh (1979) type estimator

MSE $(\bar{y}_w)$ - MSE $\left(\bar{y}^P_{\sqrt{w}}\right) \geq 0$ if

$$\min\left(2, \frac{4\rho C_Y}{W C_X} - 2\right) \leq K \leq \max\left(2, \frac{4\rho C_Y}{W C_X} - 2\right) \quad (21)$$

Thus, proposed class of estimator is more efficient than walsh (1979) estimator if condition (21) is satisfied.

Proposed class of estimator is compared with Ray et al (1979) estimator

MSE $(\bar{y}_{RS}) -$ MSE $\left(\bar{y}^P_{\sqrt{RS}}\right) \geq 0$ if

$$\min\left(-2\,,\ 2 - \frac{4\rho C_Y}{\gamma C_X}\right) \leq K \leq \max\left(-2\,,\ 2 - \frac{4\rho C_Y}{\gamma C_X}\right) \quad (22)$$

Thus Proposed class of estimator is more efficient than Ray et al (1979) estimator if condition (22) is satisfied.

Proposed class of estimator is compared with Srivenkataramana and Tracy (1979) estimator

MSE $(\bar{y}_{ST}) -$ MSE $\left(\bar{y}^P_{\sqrt{ST}}\right) \geq 0$ if

$$\min\left(-2, \frac{4\rho C_Y}{\left(1-\frac{n}{N}\right)C_X} + 2\right) \leq K \leq \max\left(-2, \frac{4\rho C_Y}{\left(1-\frac{n}{N}\right)C_X} + 2\right) \quad (23)$$

Thus Proposed class of estimator is more efficient than Srivenkataramana and Tracy (1979) estimator if condition (23) is satisfied.

Proposed class of estimator is compared with Srivenkataramana (1980) duel to ratio estimator

MSE $(\bar{y}_S) -$ MSE $\left(\bar{y}^P_{\sqrt{S}}\right) \geq 0$ if

$$\min\left(-2\,,\ 2 - \frac{4\rho C_Y}{\left(\frac{n}{N-n}\right)C_X}\right) \leq K \leq \max\left(-2\,,\ 2 - \frac{4\rho C_Y}{\left(\frac{n}{N-n}\right)C_X}\right) \quad (24)$$

Thus Proposed class of estimator is more efficient than Srivenkataramana (1980) estimator if condition (24) is satisfied.

Proposed class of estimator is compared with Vos (1980) estimator

MSE $(\bar{y}_V)$ - MSE $\left(\bar{y}^P_{\sqrt{V}}\right) \geq 0$ if

$$\min\left(-2,\ \frac{4\rho C_Y}{(1-W)C_X}+2\right) \le K \le \max\left(-2,\ \frac{4\rho C_Y}{(1-W)C_X}+2\right) \quad (25)$$

Thus Proposed class of estimator is more efficient than Vos (1980) estimator if condition (25) is satisfied.

VIII. Empirical Comparison

In order to investigate the performance of proposed class of estimator , three real population data sets have been considered and description of populations have been given in Table I. Population I has high positive correlation whereas population II has low but positive while population III has high negative correlation between auxiliary variable and study variable . The percent relative efficiency of proposed class of estimator has been computed with respect to sample mean estimator for different value of α and K and given in Table II

Table I

Parameters	Population I Source: Cochran 1977,Page186 Y:Total no. of members X:No. of children	Population II Source: Cochran 1977,Page 34 Y:Food cost X:Family size	Population III Source: US Environmental Protection agency 1991 Y:Average mile per gallon X:Engine horsepower
N	21	33	80
n	5	5	5
$\bar{Y}$	3.809524	27.4909091	33.5175
$\bar{X}$	1.714286	72.5454545	118.1625
C_Y	0.34	0.3685139	0.28767
C_X	0.72329	0.1458031	0.48045
ρ_{XY}	0.97413	0.25216031	-0.801

XI. CONCLUSION

From expressions of the bias of proposed estimator and other considered estimators, theoretically it is established that bias of proposed estimator is always half for any value of α and k. From Table II it is observed that for population I as -1.0 $\le \alpha \le$ -0.2 the efficiency of proposed estimator decreases as k increases from -1.0 to 2.0. Also, 0 $\le \alpha \le$ 0.4 the efficiency of proposed estimator increases as k increases from -1.0 to 2.0 but for 0.4 $\le \alpha \le$ 1.0 efficiency of proposed estimator first increases then decreases -1.0 $\le k \le$ 2.0. For population II as -1.0 $\le \alpha \le$ 0.2 the efficiency of proposed estimator decreases as k increases from -1.0 to 2.0 but for 0 $\le \alpha \le$ 1.0 efficiency of the proposed estimator increases as k increases from -1.0 to 2.0 . For population III as α -1.0 $\le \alpha \le$ 0.2 efficiency of the proposed estimator first increases and then decreases as k -1.0 $\le k \le$ 2.0. but for 0 $\le \alpha \le$ 1.0 contrary to population II efficiency of the proposed estimator decreases as k increases from -1.0 to 2.0. Thus it is concluded that efficiency of the proposed estimator also depends on the nature of the population.

It is also observed that the particular cases of proposed class of estimator, such as $\bar{y}_{\sqrt{W}}$, , $\bar{y}_{\sqrt{RS}}$, $\bar{y}_{\sqrt{ST}}$, $\bar{y}_{\sqrt{V}}$ and $\bar{y}_{\sqrt{S}}$ are always in general more efficient than estimators suggested by Walsh [9] ,Ray et al [10] Srivenkataramana and Tracy [11] ,Vos [12], and Srivenkataramana [14] .

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ISSN 2229-5518

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Table II: Population I

K	α= - 1	α= -0.8	α=-0.6	α=-0.4	α=-0.2	α=0	α=0.2	α=0.4	α=0.6	α=0.8	α= 1.0
-1.0	1692.43	1509.45	610.05	284.01	158.53	100	68.51	49.75	37.73	29.57	23.79
-0.8	1509.45	729.15	375.98	220.89	143.39	100	73.50	56.21	44.34	35.85	29.57
-0.6	610.05	375.98	249.62	176.11	130.27	100	79.05	64.00	52.83	44.33	37.73
-0.4	284.01	220.89	176.11	143.39	118.84	100	85.25	73.50	64.00	56.21	49.75
-0.2	158.53	143.39	130.27	118.84	108.82	100	92.19	85.25	79.05	73.50	68.51
0.0	100	100	100	100	100	100	100	100	100	100	100
0.2	68.51	73.50	79.05	85.25	92.19	100	108.82	118.84	130.27	143.39	158.53
0.4	49.75	56.21	64.00	73.50	85.25	100	118.84	143.39	176.11	220.89	284.01
0.6	37.73	44.34	52.83	64.00	79.05	100	130.27	176.11	249.62	375.98	610.05
0.8	29.57	35.85	44.34	56.21	73.50	100	143.39	220.89	375.98	729.15	1509.45
1.0	23.79	29.57	37.73	49.75	68.51	100	158.53	284.01	610.05	1509.45	1692.43
1.2	19.55	24.80	32.48	44.34	64.00	100	176.11	375.98	1058.68	1876.93	702.08
1.4	16.34	21.10	28.26	39.76	59.92	100	196.67	514.62	1736.84	1017.98	316.16
1.6	13.86	18.16	24.80	35.85	56.21	100	220.89	729.15	1876.93	497.22	172.22
1.8	11.91	15.80	21.94	32.48	52.83	100	249.62	1058.68	1226.02	276.30	106.89
2.0	10.34	13.86	19.55	29.57	49.75	100	284.01	1509.45	702.08	172.22	72.41

POPULATION II

K	α= -1	α=-0.8	α=-0.6	α=-0.4	α=-0.2	α=0	α=0.2	α=0.4	α=0.6	α=0.8	α=1
-1.0	106.45	105.79	104.80	103.49	101.87	100	97.89	95.59	93.11	90.51	87.80
-0.8	105.79	105.02	104.04	102.87	101.52	100	98.33	96.53	94.61	92.60	90.51
-0.6	104.80	104.04	103.18	102.22	101.15	100	98.76	97.45	96.06	94.61	93.11
-0.4	103.48	102.87	102.22	101.52	100.78	100	99.18	98.33	97.45	96.53	95.59
-0.2	101.87	101.52	101.15	100.78	100.40	100	99.60	99.18	98.76	98.33	97.89
0.0	100	100	100	100	100	100	100	100	100	100	100
0.2	97.89	98.33	98.76	99.18	99.60	100	100.39	100.78	101.15	101.51	101.87
0.4	95.59	96.53	97.45	98.33	99.18	100	100.78	101.52	102.22	102.87	103.48
0.6	93.11	94.61	96.06	97.45	98.76	100	101.15	102.22	103.18	104.04	104.80
0.8	90.51	92.60	94.61	96.53	98.33	100	101.52	102.87	104.04	105.02	105.79
1.0	87.80	90.51	93.11	95.59	97.89	100	101.87	103.48	104.80	105.79	106.45
1.2	85.03	88.35	91.56	94.61	97.45	100	102.22	104.04	105.43	106.35	106.76
1.4	82.21	86.14	89.97	93.62	96.99	100	102.55	104.56	105.95	106.68	106.72
1.6	79.38	83.91	88.35	92.60	96.53	100	102.87	105.02	106.35	106.79	106.32
1.8	76.55	81.64	86.70	91.56	96.06	100	103.18	105.43	106.62	106.67	105.57
2.0	73.74	79.38	85.03	90.51	95.59	100	103.48	105.79	106.76	106.32	104.49

POPULATION III

K	α=-1	α=-0.8	α=-0.6	α=-0.4	α=-0.2	α= 0	α=0.2	α=0.4	α=0.6	α=0.8	α= 1
-1.0	32.95	39.74	48.69	60.73	77.19	100	131.52	173.47	223.03	265.90	278.12
-0.8	39.74	46.69	55.47	66.69	81.17	100	124.41	155.44	192.85	232.85	265.90
-0.6	48.69	55.47	63.61	73.46	85.43	100	117.71	139.06	164.26	192.85	223.03
-0.4	60.73	66.69	73.46	81.17	89.97	100	111.43	124.41	139.06	155.44	173.47
-0.2	77.19	81.17	85.43	89.97	94.82	100	105.53	111.43	117.71	124.41	131.52
0.0	100	100	100	100	100	100	100	100	100	100	100
0.2	131.52	124.41	117.71	111.43	105.53	100	94.82	89.97	85.43	81.17	77.19
0.4	173.47	155.44	139.06	124.41	111.43	100	89.97	81.17	73.46	66.69	60.73
0.6	223.03	192.85	164.26	139.06	117.71	100	85.43	73.46	63.61	55.47	48.69
0.8	265.90	232.85	192.85	155.44	124.41	100	81.17	66.69	55.47	46.69	39.74
1.0	278.12	265.91	223.03	173.47	131.52	100	77.19	60.73	48.69	39.74	32.95
1.2	250.73	279.02	251.07	192.85	139.06	100	73.46	55.47	43.02	34.16	27.70
1.4	202.47	265.65	271.51	212.97	147.04	100	69.97	50.82	38.23	29.65	23.59
1.6	155.10	232.47	279.02	232.85	155.44	100	66.69	46.69	34.16	25.94	20.30
1.8	117.45	192.45	271.31	251.07	164.26	100	63.61	43.02	30.69	22.87	17.64
2.0	89.782	155.10	250.73	265.91	173.47	100	60.73	39.74	27.70	20.30	15.47